Random Item Effects Modeling (Part 1)

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Random Item Effects Modeling

Introduction Bayesian Item Response Modeling

Conditional Modeling

Example

Multilevel IRT

Modeling Item Responses

Collection of responses, $i = 1, \ldots, N$ persons $k = 1, \ldots, K$ items. Data matrix $N \times K$ responses:

- $$\mathbf{Y} = \begin{bmatrix} 1 & 0 & 1 & \ldots & Y_{1K} \\ 0 & 1 & 1 & \ldots & Y_{2K} \\ \vdots & & & \ddots & \vdots \\ Y_{N1} & 0 & 1 & \ldots & Y_{NK} \end{bmatrix}$$

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- Model the structure of the data
- Structure: persons (rows) and items (columns).

A Likelihood Model; Binary Responses

$$P(Y_{ik} = 1) = F(\theta_i - b_k)$$

- Transform this probability to a continuous scale of all real numbers

A Likelihood Model; Binary Responses

$$\log\left(\frac{P(Y_{ik}=1)}{1-P(Y_{ik}=1)}\right)=\theta_i-b_k$$

- The log-odd is defined on $(-\infty,\infty)$

A Likelihood Model; Binary Responses

$$\log\left(\frac{P(Y_{ik}=1)}{1-P(Y_{ik}=1)}\right)=\theta_i-b_k$$

- The Logistic Model
- The Probit Model

A Likelihood Model; Binary Responses

$$\log\left(\frac{P(Y_{ik}=1)}{1-P(Y_{ik}=1)}\right)=\theta_i+b_k$$

- The Rasch model, or One-parameter (likelihood) model.

Conditional Modeling Framework

$$P\left(Y_{ik} = 1 \mid \theta_i, b_k\right) = \frac{\exp\left(\theta_i - b_k\right)}{1 + \exp\left(\theta_i - b_k\right)}$$

Features of this model:

- Distinct parameters of persons and items

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Features of this model:

- Distinct parameters of persons and items
- Continuous latent variable $\theta \in \mathbf{R}$
- Monotonic increasing function of θ

Conditional Independence (1)

- Conditional on person parameter, item responses are assumed to be independent. (within-person dependency)

$$p\left(\mathbf{y}_i \mid \theta_i, \mathbf{b}\right) = \prod_{k=1}^{K} p\left(y_{ik} \mid \theta_i, b_k\right)$$

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- The person parameter explains (within-person) associations between item responses

Conditional Independence (2)

- Conditional on item difficulty, item responses are assumed to be independent

$$p\left(\mathbf{y}_k \mid \boldsymbol{\theta}, b_k\right) = \prod_{i=1}^{N} p\left(y_{ik} \mid \theta_i, b_k\right)$$

Conditional Independence (2)

- Conditional on item difficulty, item responses are assumed to be independent

$$p\left(\mathbf{y}_k \mid \boldsymbol{\theta}, b_k\right) = \prod_{i=1}^{N} p\left(y_{ik} \mid \theta_i, b_k\right)$$

- The item parameter explains (structural) relationships between item responses

Define Prior Model (Level-1 Parameters)

- Prior model for person parameters

$$p(\theta_i \mid \mu_\theta, \sigma_\theta^2) \quad \sim \quad \mathcal{N}\left(\mu_\theta, \sigma_\theta^2\right)$$

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- Prior model for item parameters

$$p(b_k \mid \mu_b, \sigma_b^2) \quad \sim \quad \mathcal{N}\left(\mu_b, \sigma_b^2\right)$$

Hierarchical Prior Model (Level-2 Parameters)

- Hyperprior for prior parameters:

$$\begin{aligned} \mu_\theta &\sim \mathcal{N}\left(\mu_0, \sigma_0^2\right) \\ \sigma_\theta^2 &\sim \mathcal{IG}\left(g_1, g_2\right) \end{aligned}$$

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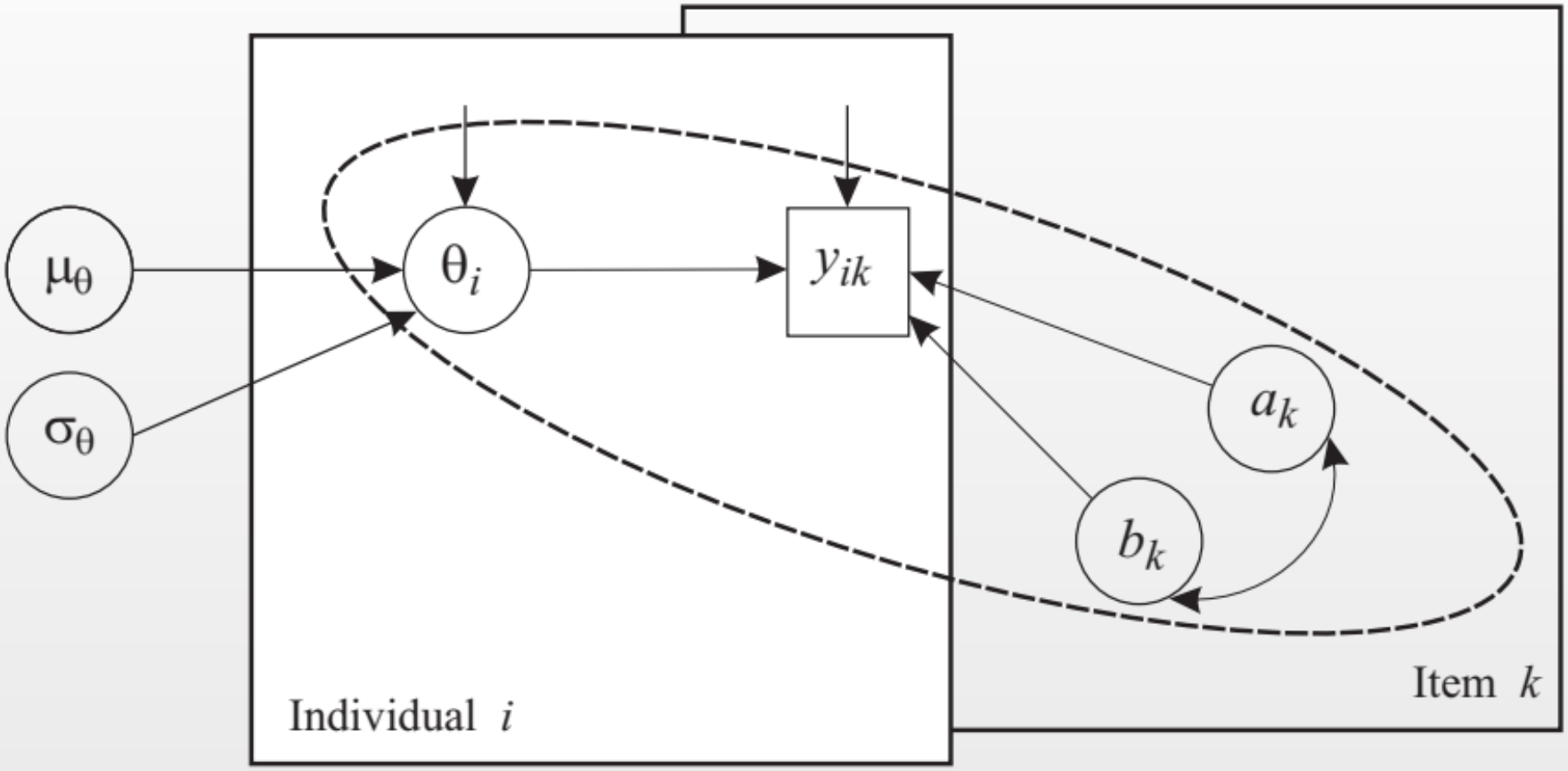
- Hyperprior model for item parameters:

$$\begin{aligned} \mu_b &\sim \mathcal{N}\left(\mu_0, \sigma_0^2\right) \\ \sigma_b^2 &\sim \mathcal{IG}\left(g_1, g_2\right) \end{aligned}$$

Two-Parameter Item Response Model



Bayesian Model Building

1. Construct a likelihood model.
2. Specify a prior population model for person parameters.
3. Specify a prior population model for item parameters.
4. Specify hyperpriors for the prior parameters:

Marginal Estimation

- Make inferences from the (marginal) posterior distributions

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$$p\left(\boldsymbol{\theta} \mid \mathbf{y}\right) = \int \int p\left(\boldsymbol{\theta} \mid \mathbf{y}, \mu_\theta, \sigma_\theta^2\right) p\left(\mu_\theta \mid \mu_0\right) p\left(\sigma_\theta^2 \mid g_1, g_2\right) d\mu d\sigma_\theta^2$$

Marginal Estimation

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- MCMC produces samples from all marginal distributions.

An Example

- Example One-Parameter Model
- R2WinBUGS interface
- Posterior inferences

WinBUGS: One-Parameter Model

Listing 1: WinBUGS Code

```
model
{
    for (i in 1:N) {
      for (k in 1:K) {
         p[i,k] <- exp(1.7*(theta[i]-b[k])) / (1+exp(1.7*(theta[i]-b[k])))
         Y[i,k] ~ dbern(p[i,k])
         }

         theta[i] ~ dnorm(0,precision)
    }

  ...
}
```

R2WinBUGS package

- Run program in WinBUGS

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- Can call WinBUGS from *R* with package R2WinBUGS

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- Can call WinBUGS from *R* with package R2WinBUGS
- Advantage: output from WinBUGS in *R*, easy to calculate some inferential statistics

The Data

- $N = 300$ persons took a math test of $K = 10$ items
- 150 group A (males), 150 group B (females)
- Focus on math achievement differences (between-persons, between-groups)

WinBUGS: One-Parameter Model

Listing 2: WinBUGS Code

```
model
{
    for (i in 1:N) {
      for (k in 1:K) {
         p[i,k] <- exp(1.7*(theta[i]-b[k])) / (1+exp(1.7*(theta[i]-b[k])))
         Y[i,k] ~ dbern(p[i,k])
         }
        meanterm[i] <- gamma * Males[i]
        theta[i] ~ dnorm(meanterm[i], precision)
    }

  ...
}
```

The Bottom Line

1. Construct a likelihood model for observed response data
2. Define prior population distributions likelihood model parameters (*hierarchical modeling*)
3. Define hyperpriors for the prior parameters
4. Use MCMC for posterior estimation and inference

Multilevel Population Model

- Respondents are grouped into larger units

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- Information characterizing respondents and higher-level units

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- Respondents are grouped into larger units
- Information characterizing respondents and higher-level units
- School effectiveness research, cross-national surveys, longitudinal survey studies

Population Model for Item Parameters

Stage 2: Prior for Item Parameters

$$(a_k, b_k)^t \sim \mathcal{N}\left(\boldsymbol{\mu}_\xi, \boldsymbol{\Sigma}_\xi\right) I_{\mathcal{A}_k}(a_k),$$

where the set $\mathcal{A}_k = \{a_k \in \mathcal{R}, a_k > 0\}$

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Stage 3: Hyper prior

$$\begin{aligned} \boldsymbol{\Sigma}_\xi &\sim \mathcal{IW}(\nu, \boldsymbol{\Sigma}_0) \\ \boldsymbol{\mu}_\xi \mid \boldsymbol{\Sigma}_\xi &\sim \mathcal{N}(\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_\xi / K_0). \end{aligned}$$

Population Model for Person Parameters

Stage 2: Prior for Person Parameters

$$
\begin{aligned}
\theta_{ij} \mid \mu_{\theta_j} &\sim \mathcal{N}\left(\mu_{\theta_j}, \sigma^2\right) \\
\mu_{\theta_j} \mid \mu_{\theta} &\sim \mathcal{N}\left(\mu_{\theta}, \tau^2\right)
\end{aligned}
$$

Population Model for Person Parameters

Stage 2: Prior for Person Parameters

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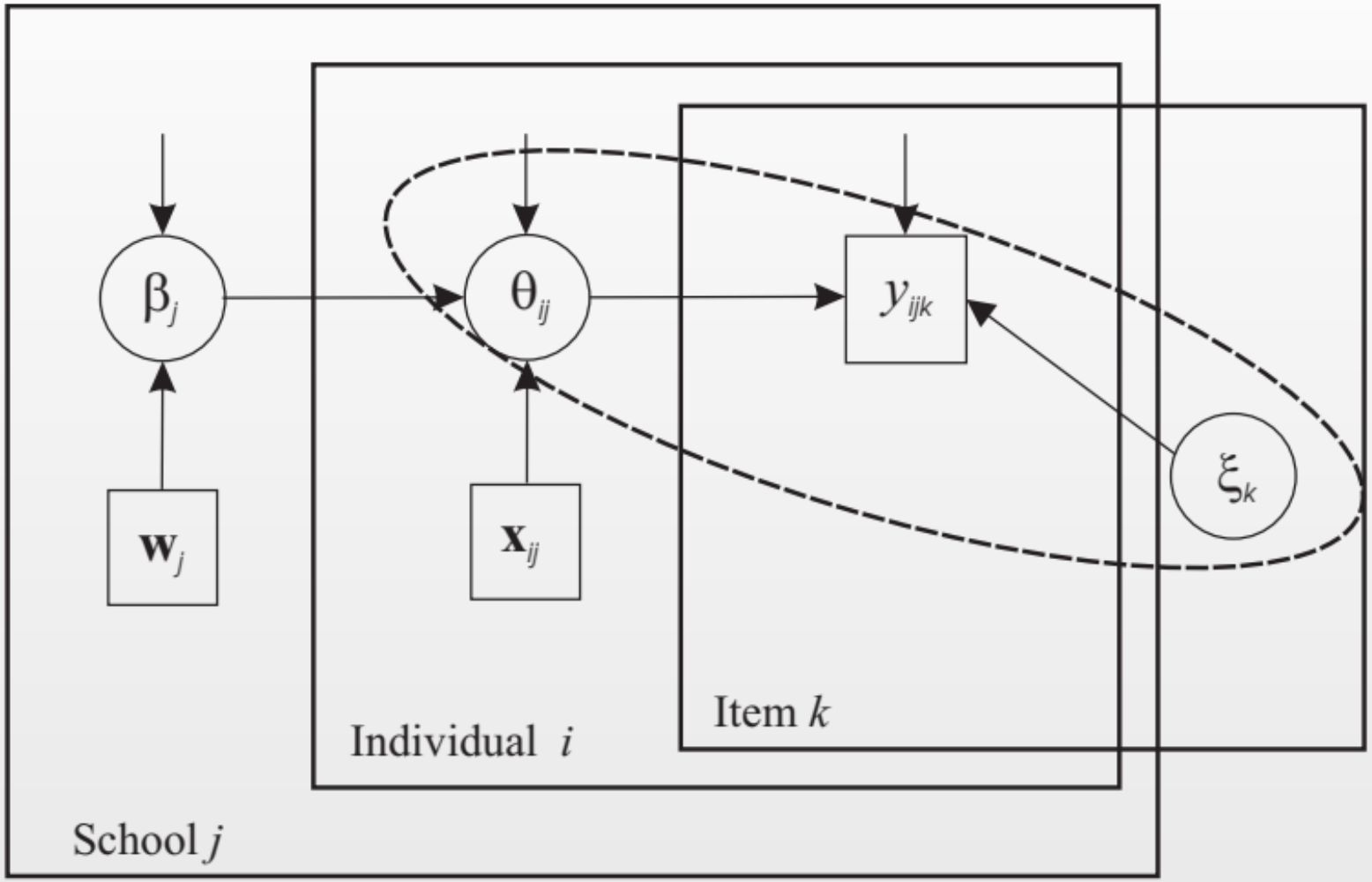
Stage 3: Hyper prior

$$\begin{aligned}\sigma^2 &\sim \mathcal{IG}\left(g_1, g_2\right) \\ \tau^2 &\sim \mathcal{IG}\left(g_1, g_2\right) \\ \mu_\theta &\sim \mathcal{N}\left(\mu_0, \sigma_0^2\right)\end{aligned}$$

Multilevel IRT (MLIRT)



The Synthesis: IRT and Multilevel Models

Stage 1 Response (Likelihood) Model

$$P\left(Y_{ijk}=1 \mid \boldsymbol{\xi}_k, \theta_{ij}\right) = \begin{cases} \Psi\left(d\left(a_k\theta_{ij}-b_k\right)\right) \\ \Phi\left(a_k\theta_{ij}-b_k\right) \end{cases}$$

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Stage 2 Structural (Prior) Model

$$\begin{aligned} \boldsymbol{\theta}_j &= \boldsymbol{x}_j\boldsymbol{\beta}_j + \boldsymbol{e}_j \\ \boldsymbol{\beta}_j &= \boldsymbol{w}_j\boldsymbol{\gamma} + \boldsymbol{u}_j \end{aligned}$$

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Stage 3 (Hyper-)Prior Model

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- Parameters are estimated from the item response data without having to condition on estimated person parameters. (Ignoring the uncertainty regarding the latent abilities may lead to biased parameter estimates)

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The Synthesis: IRT and Multilevel Models

- Parameters are estimated from the item response data without having to condition on estimated person parameters. (Ignoring the uncertainty regarding the latent abilities may lead to biased parameter estimates)
- Allows the incorporation of explanatory variables at different levels of hierarchy. (using collateral information to sample examinees and to assign items to them)
- Handle incomplete data in a very flexible way. (Item responses and subjects are missing for completely random reasons)

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- MLIRT modeling framework provides shrinkage estimators that are based on an efficient combination of the response data and the collateral information.

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$$p\left(\theta_{ij} \mid \boldsymbol{y}, \boldsymbol{x}_j, \boldsymbol{\beta}_j, \boldsymbol{\xi}, \sigma_\theta^2\right) \quad \propto \quad p\left(\boldsymbol{y} \mid \boldsymbol{\xi}, \theta_{ij}\right) p\left(\theta_{ij} \mid \boldsymbol{x}_j, \boldsymbol{\beta}_j, \sigma_\theta^2\right)$$

The Synthesis: IRT and Multilevel Models

The use of explanatory information may lead to more accurate item parameter estimates.

$$p\left(\boldsymbol{\xi}_k \mid \boldsymbol{y}, \boldsymbol{x}\right) = \int p\left(\boldsymbol{\xi}_k \mid \boldsymbol{y}, \boldsymbol{\theta}\right) p\left(\boldsymbol{\theta}; \boldsymbol{\beta}, \sigma_\theta^2, \boldsymbol{x}\right) d\boldsymbol{\theta}$$

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MLIRT for Longitudinal Item Response Data

- MLIRT model: compound symmetry model - (co)variances are equal across time

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\theta_{ij} &= \beta_{0i} + e_{ij} \\
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  5. Estimate subject-specific change across time (average change)
- Extensions to model covariance patterns

Mixture MLIRT Modeling

Modeling of asymmetrical data: Define latent groups g_1 and g_2

$$p\left(\theta_{ij} \mid \mathbf{\Omega}\right) = \sum_{g=1}^{2} \pi_{ig} p\left(\theta_{ij} \mid \mathbf{\Omega}_g\right)$$

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$$P\left(G_i = 1 \mid \boldsymbol{y}_i, \boldsymbol{\theta}_i\right) = \frac{\pi_{i1} \prod_{j=1}^{n_i} p\left(\boldsymbol{y}_{ij} \mid \theta_{ij}\right) p\left(\theta_{ij} \mid \boldsymbol{\Omega}_1\right)}{\sum_{g=1,2} \pi_{ig} \prod_{j=1}^{n_i} p\left(\boldsymbol{y}_{ij} \mid \theta_{ij}\right) p\left(\theta_{ij} \mid \boldsymbol{\Omega}_g\right)}$$