Joint Modeling of Feedback-Use and Time Data

Advances in Bayesian Item Response Modeling

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Overview

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Formative Computer-Based Assessment

- Two-stage testing: Ability - feedback use
- Observe response times (speed) and feedback times (reading)
- Dutch study: Differential use of feedback in test assessment

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Bayesian Modeling of Multivariate Count Data

A Bayesian Modeling Approach:

- Hierarchical Structured Data, uncertainty/sampling error at different levels
- Use Powerful Simulation Techniques
- Use Prior Knowledge

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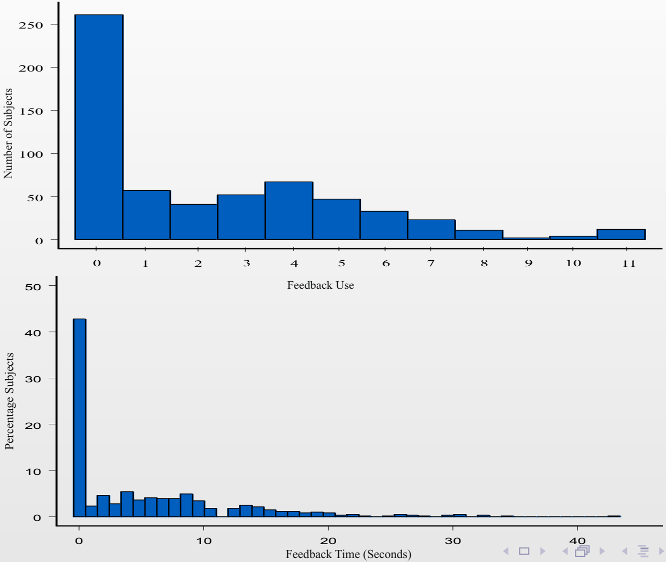
Feedback-Use and Feedback-Time Data



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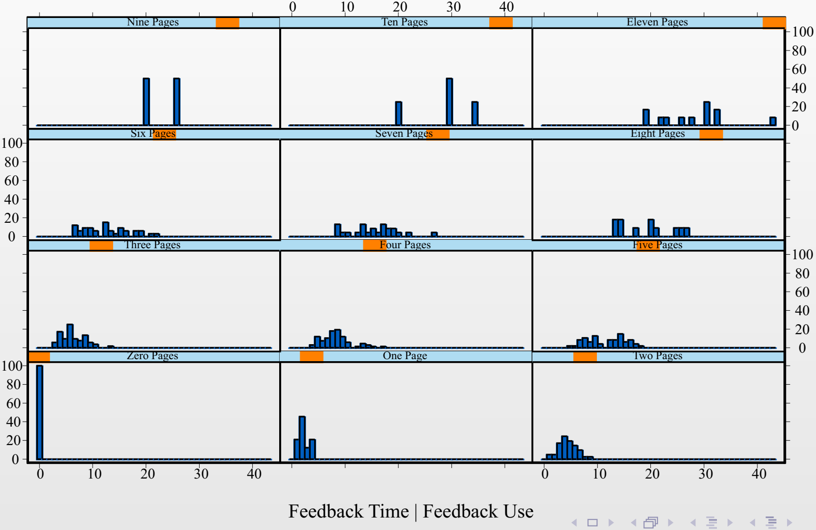

Modeling Multivariate Count Data

Count Data

	No. Pages	Total Times
Subjects	2	7
	0	0
	$\vdots$	$\vdots$
	y_i^f	y_i^t

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Summary Statistics

	Mean	SD	% Zeros	Mean | No Zeros
Feedback Use	2.35	5.35	.43	4.11
Feedback Times	2.75	6.19	.43	9.35

Overview

Feedback-Use No. Pages

The idea is to model feedback use (yes or no), feedback pages (count pages), feedback times (count seconds)

Mixture of Observed Feedback Pages

$$Y_i^f \sim \begin{cases} 0, & \text{with probability } 1-\phi_i \\ Poisson\left(\lambda_i^{(f)}\right), & \text{with probability } \phi_i, \end{cases}$$

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Model Feedback Count Data

$$\begin{aligned} P\left(Y_i^f = 0 \mid \lambda_i = \lambda_i^{(f)}\right) &= (1-\phi_i) + \phi_i e^{-\lambda_i} \\ P\left(Y_i^f = j \mid \lambda_i = \lambda_i^{(f)}\right) &= \phi_i \frac{e^{-\lambda_i}\lambda_i^j}{j!}, \end{aligned}$$

Feedback Times

Mixture of Observed Feedback Times

$$T_i^f \sim \begin{cases} 0, & \text{with probability } 1-\phi_i \\ Poisson\left(\lambda_i^{(t)}\right), & \text{with probability } \phi_i, \end{cases}$$

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Model Feedback Time Count Data

$$\begin{aligned} P\left(T_i^f = 0 \mid \lambda_i = \lambda_i^{(t)}\right) &= (1-\phi_i) + \phi_i e^{-\lambda_i} \\ P\left(T_i^f = j \mid \lambda_i = \lambda_i^{(t)}\right) &= \phi_i \frac{e^{-\lambda_i}\lambda_i^j}{j!}, \end{aligned}$$

Feedback Use

Identify (non-)users of feedback pages using explanatory subject information

Observed Feedback Use

$$Z_i \mid \lambda_i^{(t)}, \lambda_i^{(f)} \quad \sim \quad \begin{cases} 0, & \text{with probability } (1-\phi_i)P\left(Y_i^f = 0, T_i^f = 0\right) \\ 1, & \text{with probability } \phi_i\left(1 - P\left(Y_i^f = 0, T_i^f = 0\right)\right) \end{cases}$$

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Feedback Use

$$\phi_i = P\left(Z_i = 1\right) = \frac{\exp\left(\mathbf{x}_i^t \boldsymbol{\alpha}\right)}{1 + \exp\left(\mathbf{x}_i^t \boldsymbol{\alpha}\right)}$$

Population Model Subjects

Respondents are sampled independently and identically distributed.

Stage 2: Prior Expected Counts

$$\begin{aligned} \log \lambda_i^{(f)} &= \mathbf{x}_i^t \boldsymbol{\beta}_f \\ \log \lambda_i^{(t)} &= \mathbf{x}_i^t \boldsymbol{\beta}_t \end{aligned}$$

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Stage 2: Multivariate Prior Expected Counts

$$\begin{pmatrix} \log \lambda_i^{(f)} \\ \log \lambda_i^{(t)} \end{pmatrix} \sim \mathcal{N}\left(\mathbf{x}\boldsymbol{\beta}, \boldsymbol{\Sigma}_\lambda\right)$$

Population Results

	Joint Model (No Predictors)		
Component	Parameter	Mean	HPD
Feedback Use (Bernoulli part)			
Use Feedback	Intercept, α_0	.30	(.13,.45)
No Feedback	$1-\phi$	.43	(.38,.46)
Feedback Behavior (Poisson part)			
No. Pages	Intercept, μ_1	3.06	(2.69,3.46)
Time	Intercept, μ_2	7.09	(6.35,7.92)
	Correlation,Σ_{12}	.20	(.13,.27)

– HPD: 95% Highest Posterior Density interval

Overview

Ability-Speed Model

Collection of Responses and Response Times, N persons and K items

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Measuring Ability

$$P\left(Y^{a}_{ik} = 1 \mid \theta_i, a_k, b_k\right) \quad = \quad \Phi(a_k\theta_i - b_k) \text{ IRT Model}$$

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Measuring Speed of Working

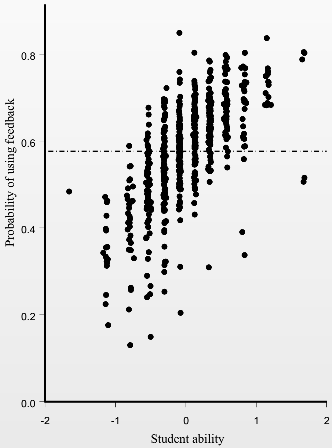
$$\log T_{ik}^{a} \mid \zeta_i, c_k, d_k \quad \sim \quad \mathcal{N}\left(d_k - c_k\zeta_i, \sigma_{\epsilon}^{2}\right) \text{ RT Model}$$

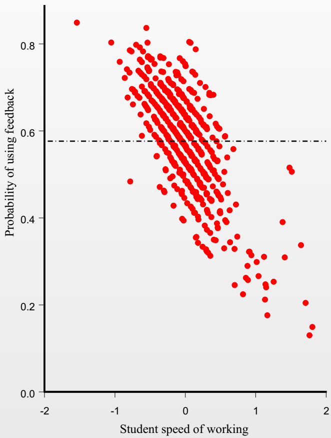
Joint Model Results

	Joint Model (Latent Predictors Speed and Ability)		
Component	Parameter	Mean	HPD
Feedback Use (Bernoulli part)			
	Intercept, α_0	.32	(.15,.47)
	$1-\phi$	.42	(.36,.48)
	Ability, α_1	.68	(.33,1.00)
	Speed, α_2	-.95	(-1.32,-.50)

– Latent predictors are grand-mean centered

Feedback-Use



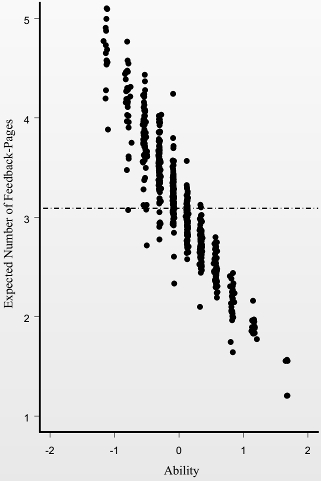


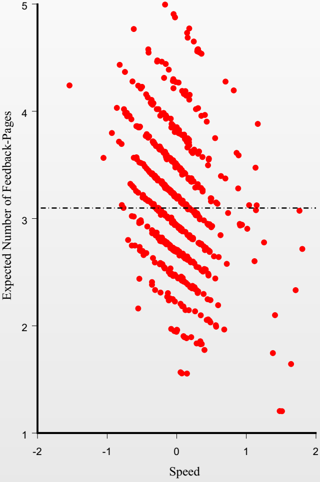
Joint Model Results

	Joint Model (latent Predictors Speed and Ability)		
Component	Parameter	Mean	HPD
Feedback Behavior (Poisson part)			
Feedback			
	Intercept, β_0	1.13 (3.09)	(1.00,1.25)
	Ability, β_1	-.40	(-.69,-.11)
	Speed, β_2	-.16	(-.52,.16)
Feedback-Time			
	Intercept, β_0	1.97 (7.17)	(1.85,2.08)
	Ability, β_1	-.33	(-.59,-.07)
	Speed, β_2	-.32	(-.63,-.03)
Correlation	Σ_{12}	.18	(.11,.24)

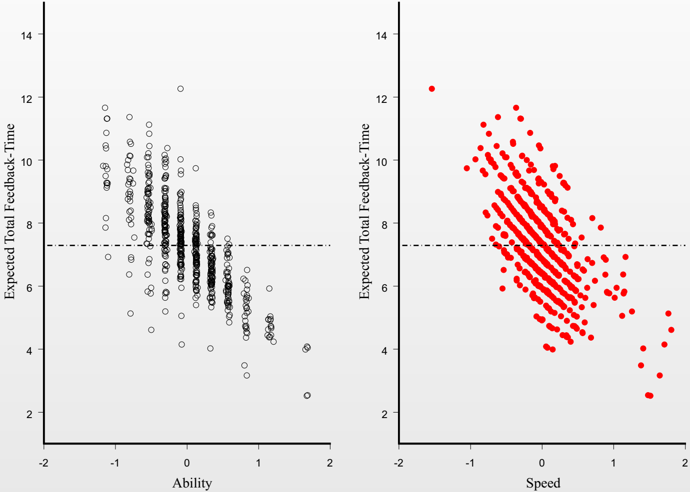
– Latent predictors are grand-mean centered

Feedback Page Counts

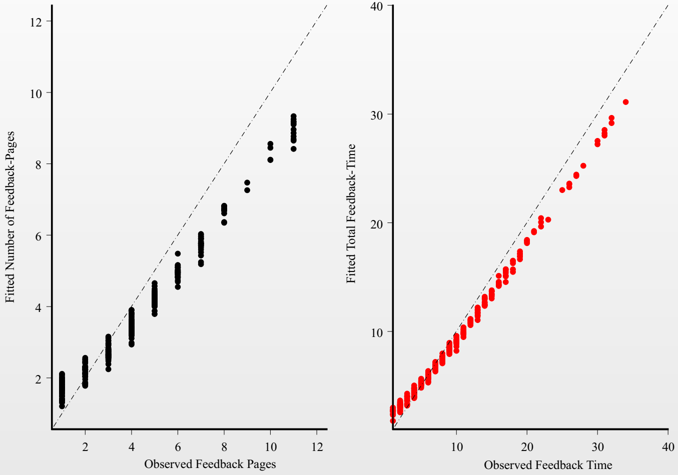




Feedback Times



Model Fit


Fitted Number of Feedback-Pages
Observed Feedback Pages
2
4
6
8
10
12
Fitted Total Feedback-Time
Observed Feedback Time
10
20
30
40

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  - Ability positively and speed negatively related to feedback use

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- Flexible joint model for multivariate zero-inflated discrete count data
- Use (higher-level) latent predictor variables
- Feedback Behavior Study
  - Heterogeneity in feedback-use versus feedback to improve learning
  - Ability positively and speed negatively related to feedback use
  - Ability and speed negatively related to feedback counts and times

Some References

- Jean-Paul Fox (2010) *Bayesian Item Response Modeling*, Springer-Science, New-York.
- Fox, J.-P., Klein Entink, R.H., van der Linden, W.J. (2007). Modeling of responses and response times with the package cirt. *Journal of Statistical Software, 20*, issue 7.
- Klein Entink, R.H., Fox, J.-P., van der Linden, W.J. (2009). A multivariate multilevel approach to the modeling of accuracy and speed of test takers. *Psychometrika, 74*, 21-48
- `www.Jean-PaulFox.com`